a course on numerical linear algebra. However, some polishing would make it more accessible to the tyro, and I hope the authors will undertake the job in revision.

G. W. S.

15[90–02].—FERENC FORGÓ, *Nonconvex Programming*, Akadémiai Kiadó, Budapest, 1988, 188 pp., $24\frac{1}{2}$ cm. Price $24.00.

The general field of finite-dimensional optimization has largely been cultivated in areas concerned with linear, integer, and convex problems, although there are also major subfields associated with stochastic, dynamic, and nondifferentiable problems.

The term "Nonconvex Programming" is generally meant to apply to optimization problems dealing with a continuous objective function and a closed constraint region (often described by continuous functions, especially linear). Such problems have traditionally been treated by algorithms developed for convex problems, often started from several different points with the hope that one will lead to a true global optimum.

There has, however, been a significant amount of work done in the last 15–20 years in the development of methods which specifically apply to problems which have proper local solutions but whose global solution is required. This book is the first to attempt a broad and extensive summary of the various aspects of those algorithms which *guarantee* to produce such global optimizers.

Integer problems, methods based on random search and unconstrained global optimization algorithms are not covered.

Chapter 1 is an especially well-written summary of the basic results of convex optimization, and Chapter 2 is devoted to the geometric notions of convex hulls and envelopes. The three basic approaches to nonconvex programming are enumeration (direct and implicit), branch and bound, and cutting plane methods. These are summarized in Chapter 3.

One of the oldest and most studied of the nonconvex problems is that of maximizing a convex function over a polytope, and there are probably a dozen distinct algorithms that have been proposed for its solution. Some of the earlier of these are detailed in Chapter 4, while Chapter 5 discusses the basic ideas involved in treating problems whose constraint region is described (at least in part) by "reverse convex constraints", i.e., inequality constraints involving convex functions which are not to be smaller than a given constant.

Chapters 6 and 7 address methods for the largest set of nonconvex problems, including the separable and quadratic varieties. The important Fixed Charge Problem is treated in Chapter 8, while the concluding part of the book collects some isolated topics such as closed form solutions and decomposition.

While there are (understandably) a number of important topics not covered, and while there are virtually no insights into the computational efficiencies of the algorithms, this book is remarkable for the sheer extent of the items covered, and the skill with which the author covers them. The list of 113 references is also outstanding.

JAMES E. FALK

Department of Operations Research
George Washington University
Washington, D. C. 20052

16[26–02, 26D15, 26B25].—P. S. BULLEN, D. S. MITRINOVIĆ & P. M. VASIĆ, *Means and Their Inequalities*, Mathematics and Its Applications (East European Series), Kluwer, Dordrecht, 1988, xix + 459 pp., $24\frac{1}{2}$ cm. Price \$89.00.

A mean of the positive values $a_1, \ldots, a_n$ is a function $F(a_1, \ldots, a_n)$ whose value lies between the smallest and largest of its arguments. Weighted means depend also on positive weights $w_1, \ldots, w_n$ with $\sum w_i = 1$. By far the most important means are the arithmetic and geometric means, $\sum w_i a_i$ and $\prod a_i^{w_i}$. These two, together with the harmonic mean $(\sum w_i a_i^{-1})^{-1}$, occupy nearly a quarter of the book under review. It is a very comprehensive survey of nearly everything that has been published on mean values, as illustrated by 52 proofs of the fundamental inequality stating that the geometric mean does not exceed the arithmetic mean. The bibliography fills 63 pages and contains approximately a thousand entries, clear evidence that the current revision of the Mathematics Subject Classification needs a better pigeonhole for means than the 1985 version provides.

Almost another quarter of the book is occupied by the power mean $(\sum w_i a_i^r)^{1/r}$, which includes the geometric mean when $r \to 0$ as well as the arithmetic and harmonic means. Its increase with r generalizes the inequality of arithmetic and geometric means, and its other properties include the inequalities of Cauchy, Hölder, and Minkowski. While the power mean is homogeneous in $a_1, \ldots, a_n$, a further generalization called the quasi-arithmetic mean, $\phi^{-1}[\sum w_i \phi(a_i)]$ with ϕ continuous and strictly monotonic, is not homogeneous unless $\phi(x)$ is a linear function of x^r or $\log x$. Other main topics are symmetric means like $[(a_1a_2 + a_1a_3 + a_2a_3)/3]^{1/2}$, means constructed in various esoteric ways, iterated means like Gauss's arithmetic-geometric mean, and finally integral means of functions. Means of operators are mentioned but not discussed. There is a list of notations and symbols and an index of authors but no subject index.

For anyone doing research on mean values or looking for an inequality between means, the book is a splendid reference work in spite of many misprints, a photographically reduced typescript with small characters, and the absence of boldface or large headings that would make it easier to navigate. One or more proofs of nearly every theorem are given expertly in a consistent notation with careful attention to conditions of equality. Above all, the reader will find many references to published papers that he would be very unlikely to discover otherwise. Because applications to other branches of mathematics are rarely mentioned, he may get the impression that inequalities for mean values have become a somewhat ingrown field of research without a strong sense of future directions. This impression will perhaps be proved incorrect by a complementary volume of applications, comments,